Solar irradiation and atmospheric irradiation

Outside surfaces receive a solar irradiation, q_{sun}, from the short wave irradiation from the sun. The irradiation from the sun is defined as short wave irradiation due to the high temperature of the surface of the sun (Wien's law). Transparent surfaces (e.g. windows) transmit and absorb this irradiation leading to a high heat flow through the transparent surface as a result of the solar irradiation. A non-transparent surface, however, does not transmit the heat but only absorbs and reflects the solar irradiation. The non-transparent surface heats up as a result of the absorbed solar irradiation. This, in turn, leads to a heat flow through the construction into the building.

The origin of solar heat gain is the direct irradiation from the sun and the diffuse irradiation from the sky through reflection and scattering on aerosols (e.g. clouds), see figure 1.

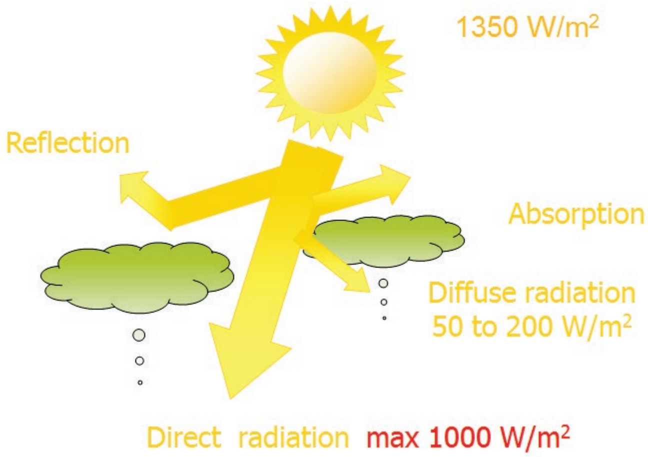


Figure 1: Diffuse and direct solar radiation [lecture slides Truus Hordijk].

The absorbed solar irradiation on building surfaces and the earth is released towards the sky. This heat flow is a long wave heat flow due to the comparatively lower temperature of the surface of the earth (Wien's law again). The temperature in outer space is approximately equal to 3 K (Evangelisti et al. 2019). The earth and its building surfaces would cool down a lot during this heat exchange with outer space due to the low temperature of the outer space. Fortunately, the earth has an atmosphere consisting of water, CO_2 and other green house gases. This atmosphere absorbs some of the infrared rays (long waves) radiated from the surface of the earth. The absorbed infrared rays raise the temperature of the atmosphere. This higher temperature of the atmosphere causes an atmospheric long wave irradiation to the earth and its buildings, q_{sky}. This is also the source of the green house effect.

In addition to the solar irradiation and the atmospheric irradiation, an outside surface can also experiences a radiation heat exchange with the surrounding built environment. The size of this heat exchange depends on the properties of the surrounding built environment. A larger, or closer, object will have a larger viewfactor[1], thus leading to a larger heat exchange with the surface than with a smaller object or an object that is further away.

[1] see Klimapedia W13 and W14 (in Dutch) or http://www.thermalradiation.net/tablecon.html

1. Solar heat gain of non-transparent surfaces

The temperature of a surface with an absorption coefficient of a_{sun} will increase through the absorption of the incoming solar irradiation q_{sun}, see figure 2. The increase in temperature of the non-transparent surface increases the irradiation leaving the surface, $q_{surface}$. The surface temperature is much lower than the temperature of the sun and is therefore long wave radiation. The long wave back radiation from the atmosphere is q_{sky}.

The following situation is considered. The outside of the non-transparent surface only 'sees' the sky dome. Six heat flows can be distinguished in this situation. The heat flows through convection (q_{conv}) and from the surface to the inside (q_i) can be found in any Building Physics textbook. The heat transfer through radiation is split into short wave radiation (incoming short wave radiation from the sun ($a_{sun}q_{sun}$) and the reflected short wave radiation from the surface ($r_{sun}q_{sun}$, not shown in figure 2) and long wave radiation (incoming long wave radiation from the sky (q_{sky}) and outgoing long wave radiation from the surface ($q_{surface}$). The reflected short wave heat flow ($r_{sun}q_{sun}$) does not heat up the surface and therefore does not play a role in the heat balance of the surface.

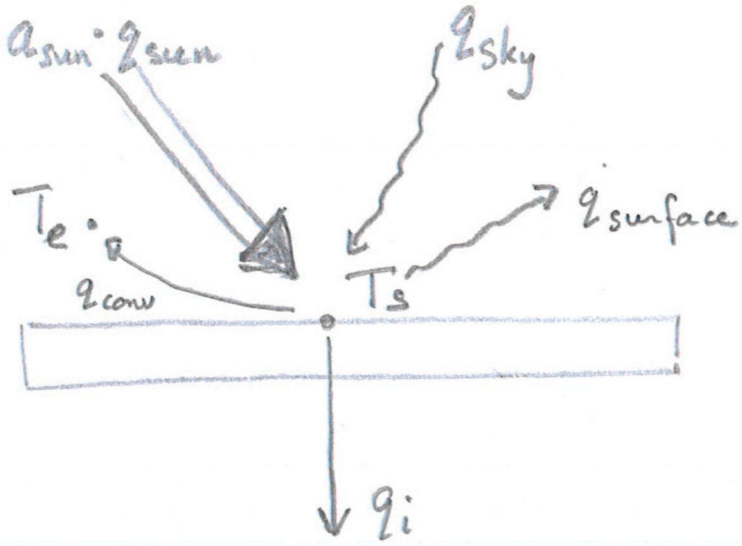


Figure 2: Heat flows between a non-transparent surface with temperature T_s and the sky.

The heat flow into the room through the non-transparent surface is q_i:

$$q_i = a_{sun} q_{sun} - q_{conv} - q_{surface} + q_{sky} \qquad 1.$$

where a_{sun}= the solar absorption of the surface [-]

2. Atmospheric irradiation.

The incoming long wave radiation from the sky (q_{sky}) has been studied by many researchers, see Evangelisti et al. (2019) for an overview.

Brunt (1) found an empirical relationship for the atmospheric back irradiation, q_{sky}, as a function of the humidity of the air expressed as the water vapour pressure, p, and the outside temperature T_e (in K):

$$q_{sky} = \sigma T_e^4 \left(a + b\sqrt{p} \right) \qquad 2.$$

For a sea climate, a = 0.55 and b = 0.005 and σ = the hemispherical Stefan-Bolzmann constant of $5{,}67 \cdot 10^{-8}$ in $W/(m^2K^4)$

This atmospheric back irradiation is lower when the outside air temperature is lower. The lower incoming energy from the sky results in a higher energy loss from the surface of a building. A higher energy loss from the surface leads to a lower surface temperature. A higher relative humidity leads to a higher amount of H_2O particles in the atmosphere. This in turn leads to more reflections from the H_2O particles and, through a higher reflected energy from the sky, to a lower net energy loss to the sky and a higher surface temperature.

More clouds, applying a similar reasoning, also lead to a higher surface temperature. This effect of clouds is taken into account by Unsworth and implemented in the NEN –EN-ISO 15927-1:2003.

In the NEN-15927, the incoming long wave radiation from the sky is given by

$$q_{sky} = \varepsilon_a \sigma T_e^4 \qquad 3.$$

with ε_a the equivalent atmospheric emittance.

Unsworth (1975) found:

$$\varepsilon_a = \varepsilon_0 \left(1 - Dc \right) + Dc \qquad 4.$$

with

$$\varepsilon_o = A + B\theta_{dp} \qquad 5.$$

where

ε_o = emissivity for clear sky conditions
c = cloud cover fraction ($0 \le c \le 1$)
θ_{dp} = dewpoint temperature measured at 2m height in a screen, in ^oC
A, D = fitted parameters from measurements
B = fitted parameter from measurements, in ^oC

If no measured data is available, the following values can be used: A = 0.745, B 0.0056 C^{-1}, D = 0.84.

If the air temperature, T_e, is measured and the cloud cover observations for low, n_L, middle, n_M, or high cloud, n_H, are known, the following equations can be used:

$$\varepsilon_o = 9.9 \cdot 10^{-6} T_e^2 \qquad 6.$$

$$\varepsilon_a = \varepsilon_0 \left[1 + a_L n_L{}^{P_L} + a_M \left(1 - n_L\right) n_M{}^{P_M} a_H + \left(1 - n_L\right)\left(1 - n_M\right) n_H{}^{p_H} \right] \qquad 7.$$

with n_L, n_M, n_H the cloud cover observations for low, n_L, middle, n_M, or high cloud, n_H.

In NEN-15927 the following values are suggested:

$$P_L = P_M = P_H = 2.5$$
$$a_L = 2.30 - 7.37 \cdot 10^{-3} \cdot \mathrm{T}_e$$
$$a_M = 2.48 - 8.23 \cdot 10^{-3} \cdot \mathrm{T}_e$$
$$a_H = 2.89 - 1.00 \cdot 10^{-2} \cdot \mathrm{T}_e$$

3. Sky Temperature

The atmospheric long-wave irradiation, q_{sky}, is the total long-wave radiation which is radiated from the atmosphere. Even though several authors use an emission coefficient in their q_{sky} equation, it is better to assume that the atmospheric irradiation has the same behaviour as the irradiation from a black body (ε_{bb} =1) with a sky temperature of T_{sky} [K][2].

$$q_{sky} = \varepsilon_{bb} \sigma T_{sky}{}^4 = \sigma T_{sky}{}^4 \qquad 8.$$

This sky temperature T_{sky} can then be calculated as:

$$T_{sky} = \sqrt[4]{\frac{q_{sky}}{\sigma}} \qquad 9.$$

or, in terms of NEN-15927 (and Unsworth) or Brunt:

$$T_{sky}{}^4 = \varepsilon_a T_e^4 \text{ or } T_{sky}{}^4 = T_e^4 \left(a + b\sqrt{p}\right) \qquad 10.$$

The vapour pressure, p, can be calculated from the relative humidity (rh) as

$$p = rh \cdot p_{sat}$$

using p_{SAT} from (NEN 15729)

[2] It is not correct to use a resulting emission coefficient with ε_{sky} when combining $q_{surface} - q_{sky}$ because q_{sky} is linear in ε_{sky} but not linear in T_{sky}.

$$p_{sat} = 6.105 \cdot e^{\left(\frac{17.269 \cdot T}{237.3+T}\right)} \text{ for } T \geq 0$$

$$p_{sat} = 6.105 \cdot e^{\left(\frac{21.875 \cdot T}{265.5+T}\right)} \text{ for } T < 0$$

or $p = rh \cdot 100 \cdot e^{\left(18.956 - \frac{4030.18}{T+235}\right)}$ with T in °C [v.d. Linden, 2018] or the equations from Tammes and Vos (1984).

A **lower** external temperature thus leads to a **lower** sky temperature. A higher amount of water, other green house gas particles, or clouds in the atmosphere lead to a higher sky temperature.

Example (Dutch climate):

The following conditions: T_e = 5 °C, relative humidities of 50 and 100 %, give sky temperatures under a clear sky as shown in table 1, according to both the Brunt and the Unsworth approximation.

	rh 50 %, p = 435 Pa	**rh 100 %, p = 870 Pa**
Brunt (1932)		
T_{sky}	250 K = -23 °C	254 K = -19 °C
$(a+b\sqrt{p})$	0.654	0.697
α_{rad}	4.2 W/m^{2}K	4.3 W/m^{2}K
$\alpha_{rad}(T_e-T_{sky})$	117 W/m^{2}K	102 W/m^{2}K
Unsworth (1975) and (NEN ISO 15927)		
T_{sky}	256 K = -17 °C	260 K =-13 °C
θ_{dp}	-4.3 °C	5.0 °C
$\varepsilon_a=\varepsilon_0$	0.72	0.77
α_{rad}	4.3 W/m^{2}K	4.4 W/m^{2}K
$\alpha_{rad}(T_e-T_{sky})$	95 W/m^{2}K	77 W/m^{2}K

Table 1: Sky temperatures at an outside temperature of 5 ℃, a relative humidity of 50 and 100 % for both the Brunt and the Unsworth approximation for a clear sky.

4. Non-transparent surface temperature at night

The heat flow into the room through the non-transparent surface was given in equation (10). At night and in other situations where the solar irradiation is absent, this amounts to:

$$q_i = -q_{conv} - q_{surface} + q_{sky} = -q_{conv} - q_{net} \qquad 11.$$

The net long-wave heat radiation from the surface to the sky is defined as q_{net}. This q_{net} is calculated in the same way as the net radiative exchange between two parallel surfaces, see W19 and W20 (in Dutch):

$$q_{net} = \varepsilon_{res}\left(\sigma T_s^4 - \sigma T_{sky}^4\right) = \varepsilon_s\left(\sigma T_s^4 - \sigma T_{sky}^4\right) \qquad 12.$$

with $\frac{1}{\varepsilon_{res}} = \frac{1}{\varepsilon_s} + \frac{1}{\varepsilon_{bb}} - 1 = \frac{1}{\varepsilon_s} + \frac{1}{1} - 1 = \frac{1}{\varepsilon_s}$ so that $\varepsilon_{res} = \varepsilon_s$.

q_{net} can be written as

$$q_{net} = \alpha_{rad}\left(T_s - T_{sky}\right) \qquad 13.$$

with

$$\alpha_{rad} = 4\varepsilon_s\sigma\left(\frac{T_s + T_{sky}}{2}\right)^3 \text{ when } \frac{T_s - T_{sky}}{T_s + T_{sky}} << 1 \qquad 14.$$

so that

$$q_i = -q_{conv} - q_{net} = -\alpha_{conv}\left(T_s - T_e\right) - \alpha_{rad}\left(T_s - T_{sky}\right) \qquad 15.$$

When the sky temperature is lower than the surface temperature, there is a heat flow from the surface to the sky. When the surface temperature is lower than the outside temperature, there is a heat flow from the outside air to the surface.

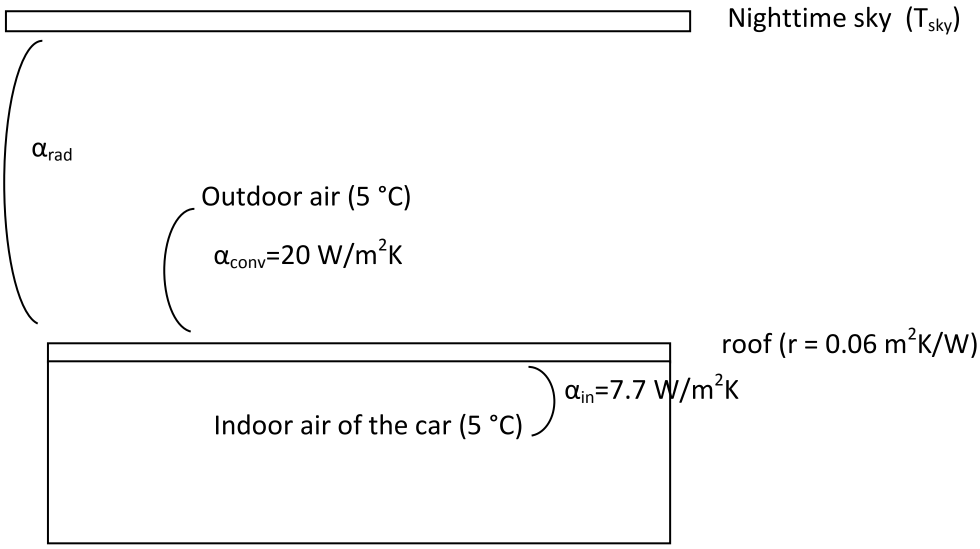
This extra radiation to the sky can results in a lower surface temperature than the air temperature, due to the radiative exchange with the sky. This, in turn, results in condensation on a surface and the risk of freezing of surfaces (car windows and icy roads) on clear sky nights.

Exercise: Nighttime Radiation

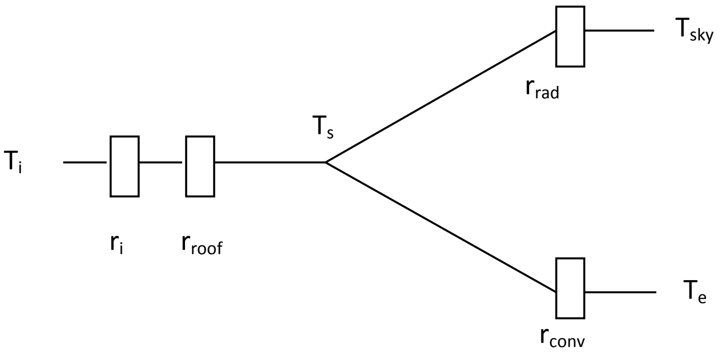
a. Why can the roof of a car be frozen in the morning when the air temperature has not been below zero during the night?

The heat exchange (radiation) with the nighttime clear sky can be so high that the resulting temperature of the car becomes less than 0 °C, even though the air temperature is above 0 °C.

b. Calculate the temperature of the roof of the car (T_s) under the assumption that the inside of the car has a temperature which equals the outdoor temperature of 5 °C. The heat transfer coefficient indoors is 7.7 W/m^{2}K, the heat transfer coefficient for convection outside is 20 W/m^{2}K (conduction can be neglected). The heat transfer coefficient between the nighttime sky and the outside of the car is α_{rad} W/m^{2}K. Use the Brunt approximation at a relative humidity outside of 80 %.



In this question the problem is that the heat transfer from the surface of the car to the outside is split in two different flows:



For the temperature of the roof of the car, we have three heat flows which should together be zero, under stationary conditions, as in equation 15:

$$q_i = -q_{conv} - q_{net} = -\alpha_{conv}\left(T_s - T_e\right) - \alpha_{rad}\left(T_s - T_{sky}\right)$$

with q_{net} describing the long wave radiative heat exchange q_{rad}.

T_{sky} can be calculated using equation 10 and making sure that the temperatures are in Kelvin:

$$T_{sky} = T_e\left(a + b\sqrt{p}\right)^{\frac{1}{4}} = (273+5)\left(0.55 + 0.005\sqrt{696}\right)^{\frac{1}{4}} = 253 \text{ K} = -20^o C$$

The vapour pressure can be calculated using $p = rh \cdot 100 \cdot e^{\left(18.956 - \frac{4030.18}{T+235}\right)}$ with T in °C, so that p = 696 Pa at an outside temperature of 5 °C and a rh = 80 % (v.d. Linden, 2018). For a sea climate we assumed a = 0.55 and b = 0.005.

q_i, q_{conv} and q_{rad} can be expressed in terms of temperatures and resistances (or heat transfer coefficients):

$$q_i = -\frac{T_s - T_{in}}{r_i + r_{roof}}$$

$$q_{rad} + q_{conv} = \frac{T_s - T_{sky}}{r_{rad}} + \frac{T_s - T_e}{r_{conv}} = \alpha_{rad}\left(T_s - T_{sky}\right) + \alpha_{conv}\left(T_s - T_e\right)$$

We do not know the heat transfer through radiation, α_{rad}, as we do not know the temperature of the surface. As a first approximation we assume that the surface temperature is the same as the outside temperature. We assume an emission coefficient of the surface of $\varepsilon_S = 0.9$. Then

$$\alpha_{rad} = 4\varepsilon_s \sigma \left(\frac{T_s + T_{sky}}{2}\right)^3 = 4 \cdot 0.9 \cdot 5.67 \cdot 10^{-8} \left(\frac{5 + 273 - 20 + 273}{2}\right)^3 = 3.8 \text{ W/m}^2 K$$

Rewriting the heat flow equations gives:

$$-\frac{\left(T_s - T_{in}\right)}{r_{roof} + \frac{1}{\alpha_{in}}} = \alpha_{rad}\left(T_s - T_{sky}\right) + \alpha_{conv}\left(T_s - T_e\right)$$

and

$$T_s\left(\frac{-1}{r_{roof} + \frac{1}{\alpha_{in}}} - \alpha_{rad} - \alpha_{conv}\right) = \frac{-T_{in}}{r_{roof} + \frac{1}{\alpha_{in}}} - \alpha_{rad} \cdot T_{sky} - \alpha_{conv} \cdot T_e$$

thus

$$T_s = \frac{\frac{-T_{in}}{r_{roof} + \frac{1}{\alpha_{in}}} - \alpha_{rad} \cdot T_{sky} - \alpha_{conv} \cdot T_e}{\left(\frac{-1}{r_{roof} + \frac{1}{\alpha_{in}}} - \alpha_{rad} - \alpha_{conv}\right)} = \frac{\left(\frac{-5}{0.06 + \frac{1}{7.7}} + 3.8 \cdot 20 - 20 \cdot 5\right)}{\left(\frac{-1}{0.06 + \frac{1}{7.7}} - 3.8 - 20\right)} = \frac{-50.33}{-29.07} = 1.7\ ^0C$$

Thus: the surface temperature of the roof is lower than the air temperature due to the extra heat exchange with the sky.

c. Is the given radiative heat transfer coefficient of 3.8 W/m²K realistic? Assume an emission coefficient of the roof of 0.9.

Using equation 14 it is possible to calculate a more accurate radiative heat transfer coefficient:

$$\alpha_{rad} = 4\varepsilon_s\sigma\left(\frac{T_s + T_{sky}}{2}\right)^3 = 4\cdot 0.9\cdot 5.67\cdot 10^{-8}\left(\frac{1.7+273-20+273}{2}\right)^3 = 3.7\ \mathrm{W/m^2}K$$

In this case the assumed radiative heat transfer is quite realistic. If this was not the case, iterating between α_{rad} and T_s will give the final result. An alternative option is to use the non-linearized equation for α_{rad} and solving the non-linearized final equation numerically , for example EXCEL.

5. Thermal radiation to the sky

The NEN 13790: 2008 gives a way to calculate the total heat loss from inside through a roof to the sky. This method is also the basis for the energy performance regulations NEN 7210 and NTA 8800. In these documents the heat loss through a roof is calculated as the sum of the heatflow between the inside and outside temperature and the extra heatflow to the sky, see figure 3 and equation 16:

$$q_i = -\frac{T_i - T_e}{R_{tot}} - q_{atmospheric\_radiation} \qquad 16.$$

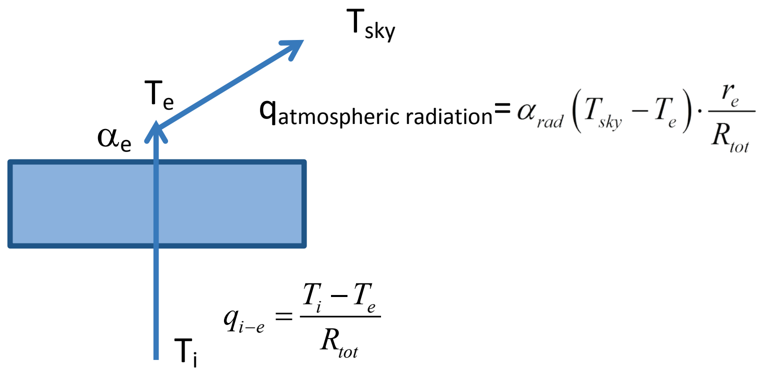


Figure 3: Separation of the total heat flow into a standard heatflow between inside and outside and a heat flow as a result of atmospheric radiation.

To obtain the equation given in NEN 13790:2008, the following mathematical steps need to be taken. given in more detail in appendix:

Creative rewriting of equation (15) gives:

$$q_i = -\alpha_{conv}\left(T_s - T_e\right) - \alpha_{rad}\left(T_s - T_{sky}\right) = -\alpha_e\left(T_s - T_e\right) - \alpha_{rad}\left(T_e - T_{sky}\right) \qquad 17.$$

with

$$\alpha_e = \alpha_{conv} + \alpha_{rad}\,.$$

The surface temperature is an unknown and can be eliminated by realising that the heat flow, q_i, through the 1-dimensional surface is constant, i.e.

$$q_i = \frac{T_s - T_i}{R_c + r_i} \qquad 18.$$

It is possible to eliminate the surface temperature from the equation by combining equations (17) and (18):

$$q_i = -\alpha_e\left(T_s - T_e\right) - \alpha_{rad}\left(T_e - T_{sky}\right) = \frac{T_s - T_i}{R_c + r_i} \qquad 19.$$

leading to

$$T_s = \frac{r_e \cdot \left(R_c + r_i\right)}{R_{tot}}\left(\frac{T_e}{r_e} + \frac{T_i}{R_c + r_i} - \frac{T_e - T_{sky}}{r_{rad}}\right) \qquad 20.$$

Filling in the surface temperature, T_S, in equation (17):

$$\begin{aligned} q_i &= -\alpha_e\left(T_s - T_e\right) - \alpha_{rad}\left(T_e - T_{sky}\right) \\ &= \frac{T_e - T_i}{R_{tot}} - \left(T_e - T_{sky}\right)\frac{r_e}{R_{tot} \cdot r_{rad}} \end{aligned} \qquad 21.$$

The extra energy to the sky, thus an energy loss from the building, is then

$$q_{atmospheric\_radiation} = \left(T_e - T_{sky}\right)\frac{r_e}{R_{tot} \cdot r_{rad}} = \alpha_{rad}\left(T_e - T_{sky}\right) \cdot \frac{r_e}{R_{tot}} \qquad 22.$$

This extra radiation consists of a radiative term, $\alpha_{rad}\left(T_e - T_{sky}\right)$, and a correction term for the insulation of the surface, $\frac{r_e}{R_{tot}}$ [3]. The radiative term for a clear sky is generally around 100 W/m^{2}K at an outside temperature of 5 °C and relative humidity of 50 and 100%, see table 1.

According to NEN 13790. when the sky temperature is not available from climatic data, the average difference between the external air temperature and the sky temperature should be taken as 9 K in sub-polar areas, 13 K in the tropics and 11 K in intermediate zones.

Acknowledgements:

This document started as a translation of W21 from klimapedia.nl by E.H. Tumbaum and J.J.M. Cauberg

[3] For very well insulated roofs, as in the Netherlands with R ≥ 6 m^{2}K/W for new buildings, the heat flow as a result of atmospheric radiation to the sky is negligible due to the low correction term.

References

NEN-EN-ISO 13790: 2008 EN, Energieprestatie van gebouwen - Berekening van het energiegebruik voor verwarming en koeling, in Dutch

NEN 7210: 2012 NL, Energieprestatie van gebouwen – Bepalingsmethode, in Dutch

NTA 8800:2019 NL, Energieprestatie van gebouwen – Bepalingsmethode, in Dutch

Brunt D., Notes on radiation in the atmosphere, Quarterly Journal of the Royal Meteorological Society, 1932, p. 389

Evangelisti et al., (2019), On the sky temperature models and their influence on buildings energy performance: A critical review, Energy and Buildings 183, 2019, 607-625

v.d. Linden et al., Building Physics 2nd edition, 2018, ThiemeMeulenhoff, Amersfoort, Netherlands, Chapter 2.

Tammes E., B.H. Vos, Warmte- en Vochttransport in Bouwconstructies, 2e dr. Kluwer Technische Boeken B.V., 1984

Tumbaum E.H. and J.J.M. Cauberg, W19, Warmteoverdracht door straling, klimapedia.nl, accessed September 17, 2019, in Dutch

Tumbaum E.H. and J.J.M. Cauberg, W20, Warmteoverdracht in een luchtgevulde spouw, klimapedia.nl, accessed September 17, 2019, in Dutch

Tumbaum E.H. and J.J.M. Cauberg, W21, Zonbestraling en atmosferische straling, klimapedia.nl, accessed September 17, 2019, in Dutch

Unworth M.H., Longwave radiance at the ground. Quarterly Journal of the Royal Meteorological Society, 101, 1975, pp.25-34

Appendix A: More detailed derivation of equations (17-22):

Creative rewriting of equation (15) gives:

$$q_i = -\alpha_{conv}\left(T_s - T_e\right) - \alpha_{rad}\left(T_s - T_{sky}\right) = -\alpha_{conv}\left(T_s - T_e\right) - \alpha_{rad}\left(T_s - T_e + T_e - T_{sky}\right)$$
$$= -\alpha_{conv}\left(T_s - T_e\right) - \alpha_{rad}\left(T_s - T_e\right) - \alpha_{rad}\left(T_e - T_{sky}\right)$$
$$= -\alpha_e\left(T_s - T_e\right) - \alpha_{rad}\left(T_e - T_{sky}\right)$$
$$q_i = -\alpha_{conv}\left(T_s - T_e\right) - \alpha_{rad}\left(T_s - T_{sky}\right) = -\alpha_{conv}\left(T_s - T_e\right) - \alpha_{rad}\left(T_s - T_e + T_e - T_{sky}\right)$$
$$= -\alpha_{conv}\left(T_s - T_e\right) - \alpha_{rad}\left(T_s - T_e\right) - \alpha_{rad}\left(T_e - T_{sky}\right)$$
$$= -\alpha_e\left(T_s - T_e\right) - \alpha_{rad}\left(T_e - T_{sky}\right) \tag{i}$$

with

$$\alpha_e = \alpha_{conv} + \alpha_{rad} .$$

The surface temperature is an unknown and can be eliminated by realising that the heat flow, q_i, through the 1-dimensional surface is constant, i.e.

$$q_i = \frac{T_s - T_i}{R_c + r_i} \tag{ii}$$

It is possible to eliminate the surface temperature from the equation by combining equations (i) and (ii):

$$q_i = -\alpha_e\left(T_s - T_e\right) - \alpha_{rad}\left(T_e - T_{sky}\right) = \frac{T_s - T_i}{R_c + r_i} \tag{iii}$$

More mathematics:

$$-\frac{\left(T_s - T_e\right)}{r_e} - \frac{\left(T_e - T_{sky}\right)}{r_{rad}} = \frac{T_s - T_i}{R_c + r_i}$$
$$T_s\left(-\frac{1}{r_e} - \frac{1}{R_c + r_i}\right) = -T_s\left(\frac{R_c + r_i + r_e}{r_e \cdot \left(R_c + r_i\right)}\right) = T_e\left(-\frac{1}{r_e}\right) - T_i\left(\frac{1}{R_c + r_i}\right) + \frac{\left(T_e - T_{sky}\right)}{r_{rad}}$$
$$-T_s = \frac{r_e \cdot \left(R_c + r_i\right)}{R_c + r_i + r_e}\left(-\frac{T_e}{r_e} - \frac{T_i}{R_c + r_i} + \frac{T_e - T_{sky}}{r_{rad}}\right)$$
$$T_s = \frac{r_e \cdot \left(R_c + r_i\right)}{R_{tot}}\left(\frac{T_e}{r_e} + \frac{T_i}{R_c + r_i} - \frac{T_e - T_{sky}}{r_{rad}}\right) \tag{iv}$$

Filling in the surface temperature, T_S, in equation (i):

$$\begin{aligned}
q_i &= -\alpha_e\left(T_s - T_e\right) - \alpha_{rad}\left(T_e - T_{sky}\right) \\
&= -\alpha_e\left(\frac{r_e\cdot\left(R_c + r_i\right)}{R_{tot}}\left(\frac{T_e}{r_e} + \frac{T_i}{R_c + r_i} - \frac{T_e - T_{sky}}{r_{rad}}\right) - T_e\right) - \alpha_{rad}\left(T_e - T_{sky}\right) \\
&= -\left(\frac{r_e\cdot\left(R_c + r_i\right)}{R_{tot}\cdot r_e}\left(\frac{T_e}{r_e} + \frac{T_i}{R_c + r_i} - \frac{T_e - T_{sky}}{r_{rad}}\right) - \frac{T_e}{r_e}\right) - \frac{T_e - T_{sky}}{r_{rad}} \\
&= -\left(\frac{\left(R_c + r_i\right)}{R_{tot}}\left(\frac{T_e}{r_e} + \frac{T_i}{R_c + r_i}\right) - \frac{T_e}{r_e}\right) + \left(T_e - T_{sky}\right)\left(-\frac{1}{r_{rad}} + \frac{\left(R_c + r_i\right)}{R_{tot}\cdot r_{rad}}\right) \\
&= -\left(\frac{\left(R_c + r_i\right)}{R_{tot}}\frac{T_e}{r_e} - \frac{T_e}{r_e}\right) - \frac{T_i}{R_{tot}} + \left(T_e - T_{sky}\right)\left(\frac{-\left(R_c + r_i + r_e\right) + \left(R_c + r_i\right)}{R_{tot}\cdot r_{rad}}\right) \\
&= -\left(T_e\frac{\left(R_c + r_i\right) - \left(R_c + r_i + r_e\right)}{R_{tot}\cdot r_e}\right) - \frac{T_i}{R_{tot}} + \left(T_e - T_{sky}\right)\left(-\frac{r_e}{R_{tot}\cdot r_{rad}}\right) \\
&= T_e\frac{r_e}{R_{tot}\cdot r_e} - \frac{T_i}{R_{tot}} + \left(T_e - T_{sky}\right)\left(-\frac{r_e}{R_{tot}\cdot r_{rad}}\right) \\
&= \frac{T_e - T_i}{R_{tot}} - \left(T_e - T_{sky}\right)\frac{r_e}{R_{tot}\cdot r_{rad}}
\end{aligned} \qquad \text{(v)}$$

The extra energy to the sky, thus an energy loss from the building, is then

$$q_{atmospheric\_radiation} = \left(T_e - T_{sky}\right)\frac{r_e}{R_{tot}\cdot r_{rad}} = \alpha_{rad}\left(T_e - T_{sky}\right)\cdot\frac{r_e}{R_{tot}} \qquad \text{(vi)}$$

Appendix B: Comparison of NEN 13790

Equation (21) is the same as the equation for the thermal radiation to the sky in NEN 13790:

$$\Phi_R = R_{se}U_c A_c h_r \Delta\theta_{er} \qquad \text{(vii)}$$

with

$$\Phi_R = q_{atmospheric\_radiation}\ A_c$$
$$R_{se} = r_e$$
$$U_c = \frac{1}{r_e + R_c + r_i} = \frac{1}{R_{tot}}$$
$$A_c = \text{projected area of the surface element}$$
$$h_r = \alpha_{rad}$$
$$\Delta\theta_{er} = T_e - T_{sky}$$