Stichting Kennisbank Bouwfysica

MEMORANDUM
WISKUNDE EN STATISTIEK

INHOUD:

1. DIFFERENTIEREN EN INTEGREREN

$f(x)$	$f'(x)=\frac{df}{dx}$	$\int f(x)dx$
ax^n	anx^{n-1}	$a\frac{x^{n+1}}{n+1}+c$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\ln x+c$
$\sin x$	$\cos x$	$-\cos x+c$
$\cos x$	$-\sin x$	$\sin x+c$
$\ln x$	$\frac{1}{x}$	$x\ln x-x+c$
e^{ax}	ae^{ax}	$\frac{1}{a}e^{ax}+c$
$u(x)\cdot v(x)$	$u(x)\cdot v'(x)+v(x)\cdot u'(x)$	
$\frac{u(x)}{v(x)}$	$\frac{u'(x)\cdot v(x)-v'(x)\cdot u(x)}{(v(x))^2}$	
$u(v(x))$	$v'(x)\cdot u'(v(x))$	

2. INTEGRAALTABEL

(2.1) $\int x^a dx = \frac{1}{a+1} x^{a+1} \qquad (a \in R \setminus \{-1\})$

(2.2) $\int \frac{dx}{x} = \ln|x|$

(2.3) $\int x^a dx = \ln|f(x)|$

(2.4) $\int x^a dx = e^x$

(2.5) $\int x^a dx = \frac{1}{\ln a} a^x \qquad (a \in R \setminus \{1\})$

(2.6) $\int \sin x \, dx = -\cos x$

(2.7) $\int \cos x \, dx = \sin x$

(2.8) $\int \frac{dx}{\sin^2 x} = -\cot x$

(2.9) $\int \frac{dx}{\cos^2 x} = \tan x$

(2.10) $\int \tan x \, dx = -\ln|\cos x|$

(2.11) $\int \frac{dx}{\sin x} = \ln\left|\tan\frac{x}{2}\right| = \ln\frac{1-\cos x}{|\sin x|} = \ln\frac{|\sin x|}{1+\cos x}$

(2.12) $\int \frac{dx}{\cos x} = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| = \ln\frac{1+\sin x}{|\cos x|} = \ln\frac{|\cos x|}{1-\sin x}$

(2.13) $\int \sin^p x \cos^q x \, dx \qquad (p,q \in N)$

p oneven, stel cos x = t

q oneven,stel sin x = t

p en q even, voer dubbele hoek in met behulp van $\cos^2 x = \frac{1+\cos 2x}{x}$ of $\sin^2 x = \frac{1-\cos 2x}{2}$

(2.14) $\int \sin mx \sin nx \, dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} \qquad (m^2 \neq n^2)$

(2.15) $\int \cos mx \cos nx \, dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} \qquad (m^2 \neq n^2)$

(2.16) $\int \sin mx \cos nx \, dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} \qquad (m^2 \neq n^2)$

(2.17) $\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

(2.18) $\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

(2.19) $\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\arctan\frac{bx}{a} \qquad (a,b \in R^+)$

(2.20) $$\int \frac{dx}{a^2 - b^2 x^2} = \frac{1}{2ab} \ln \left| \frac{a + bx}{a - bx} \right| \qquad \left(a, b \in R^+\right)$$

(2.21) $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} \qquad \left(a \in R^+\right)$$

(2.22) $$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) \qquad \left(a \in R^+\right)$$

(2.23) $$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left|x + \sqrt{x^2 - a^2}\right| \qquad \left(a \in R^+\right)$$

(2.24) $$\int \sqrt{a^2 - x^2}\, dx \qquad \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} \qquad \left(a \in R^+\right)$$

(2.25) $$\int \sqrt{a^2 + x^2}\, dx \qquad \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2} \ln\left(x + \sqrt{a^2 + x^2}\right) \qquad \left(a \in R^+\right)$$

(2.26) $$\int \sinh x dx = \cosh x$$

(2.27) $$\int \cosh x dx = \sinh x$$

(2.28) $$\int_0^{\frac{\pi}{2}} \sqrt{a^2 - x^2}\, dx = \frac{\pi a^2}{4} \qquad \left(a \in R^+\right)$$

(2.29) $$\int_0^{\frac{\pi}{2}} \sin^p x dx = \int_0^{\frac{\pi}{2}} \cos^p x dx$$
$$= \frac{\text{p - 1}}{\text{p}} \frac{p-3}{p-2} \cdots \frac{4}{5} \frac{2}{3} \text{ voor p} = \text{oneven} \geq 3$$
$$= \frac{\text{p - 1}}{\text{p}} \frac{p-3}{p-2} \cdots \frac{3}{4} \frac{1}{2} \frac{\pi}{2} \text{ voor p} = \text{oneven} \geq 2$$

(2.30) $$\int_0^{\infty} e^{-x} x^n dx = \text{n!} \qquad \left(n \in N\right)$$

(2.31) $$\int_0^{\infty} x^n (1 - x)^m dx = \frac{(\text{n!})(\text{m!})}{(\text{m} + \text{n} + 1)!} \qquad \left(n, m \in N\right)$$

3. GONIOMETRISCHE BETREKKINGEN

$\sin^2 \alpha + \cos^2 \alpha = 1$

$\sin (\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$\sin (\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

$\cos (\alpha + \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$\cos (\alpha - \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$\sin \alpha + \sin \beta = 2 \sin ½ (\alpha + \beta) \cos ½ (\alpha - \beta)$

$\sin \alpha - \sin \beta = 2 \cos ½ (\alpha + \beta) \sin ½ (\alpha – \beta)$

$\cos \alpha + \cos \beta = 2 \cos ½ (\alpha + \beta) \cos ½ (\alpha - \beta)$

$\cos \alpha - \cos \beta = 2 \sin ½ (\alpha + \beta) \sin ½ (\alpha - \beta)$

Driehoeken

sinusregel: $\frac{a}{\sin \alpha} = \frac{a}{\sin \beta} = \frac{a}{\sin \gamma}$.

cosinusregel: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Cirkel, bol, cilinder

Cirkel:	omtrek	$2\pi r$	oppervlakte	πr^2
Bol:	oppervlakte	$4\pi r^2$	inhoud	$4/3\ \pi\ r^3$
Cilinder:	oppervlakte	$2\pi rh$	inhoud	$\pi r^2 h$

4. LOGARITMISCHE BETREKKINGEN

Natuurlijke logaritmen

$$\ln x = \int_1^x \frac{dt}{t}$$

$$\ln x = {}^e\log x$$

$$\ln e = 1$$

Logaritmen, algemeen

$${}^g\log x = y \quad \Leftrightarrow \quad g^y = x$$

$$\log ab = \log a + \log b$$

$$\log \frac{a}{b} = \log a - \log b$$

$$\log a^b = b \log a$$

$$a^x = g^{\left(x\ {}^g\log a\right)} \quad \text{voor elk grondgetal g}$$

Exponentiële functie

$$\exp 0 = 1$$

$$\exp(a+b) = \exp a \cdot \exp b$$

$$\exp(a-b) = \frac{\exp a}{\exp b}$$

$$a^x = \exp(x \ln a)$$

5. WISKUNDIGE NOTATIES

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + ... + a_n$$

$$\prod_{i=1}^{n} a_i = a_1 \cdot a_2 \cdot a_3 \cdot ... \cdot a_n$$

n! (*n* faculteit) = 1 · 2 · 3 · … · (n - 1) · n

$\left(\frac{n}{k}\right)$ binomiaalcoëfficiënt $= \frac{n!}{k!(n-k)!}$

6. RIJEN EN REEKSEN

Rekenkundige rij:

$$\sum_{i=1}^{n}\left(a\left(i-1\right)v\right)=\frac{1}{2}n\left(2a+\left(n-1\right)v\right)$$

Meetkundige rij:

$$\sum_{i=1}^{n}a\,r^{i-1}=a\frac{r^{n}-1}{r-1}$$

$$\sum_{i=1}^{\infty}a\,x^{i-1}=\frac{a}{1-x}\quad \text{als -1 < x < 1}$$

Binomium van Newton:

$$\left(a+b\right)^{n}=\sum_{k=0}^{n}\left(\frac{n}{k}\right)a^{n-k}b^{k}$$

Formule van Taylor:

$$f\left(a+h\right)=f\left(a\right)+hf'\left(a\right)+\frac{h^{2}}{2!}f''\left(a\right)+\ldots+\frac{h^{n}}{n!}f^{(n)}\left(a+\vartheta h\right)$$

met $\vartheta\in\left(0,1\right)$

Formule van McLaurin:

$$f\left(x\right)=f\left(0\right)+xf'\left(0\right)+\frac{x^{2}}{2!}f''\left(0\right)+\ldots+\frac{h^{n}}{n!}f^{(n)}\left(\xi\right)$$

met $0\le\xi\le x$

$$e^{x}=1+x+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}+\ldots$$

$$\sin x=x-\frac{x^{3}}{3!}+\frac{x^{5}}{5!}-\ldots$$

$$\cos x=1-\frac{x^{2}}{2!}+\frac{x^{4}}{4!}-\ldots$$

$$\ln\left(x+1\right)=x-\frac{x^{2}}{2}+\frac{x^{3}}{3}-\ldots\quad \text{als -1 < x < 1}$$

$$\left(1\pm\alpha\right)^{n}=1\pm n\alpha\quad \text{voor}\,|\,\alpha\,|<<1$$

7. STATISTIEK

Discrete stochastische variabale:

Verwachting:
$$E(\underline{x}) = \sum_x x\, P(x)$$

Variantie:
$$\mathrm{Var}(\underline{x}) = \mathrm{E}\left((x - \mathrm{E}(\underline{x}))^2\right)$$

Standaardafwijking (deviatie):
$$\sigma(\underline{x}) = \sqrt{\mathrm{Var}(\underline{x})}$$

Continue stochastische variabele:

Verwachting:
$$E(\underline{x}) = \int_{-\infty}^{\infty} x f_{\underline{x}}(x)\,dx$$
met $f_{\underline{x}}(x)$ is de kansdichtheidsfunctie

Bijzondere verdelingen:

Binomiaalverdeling:
Kans op *x* successen in een binomiaal kansexperiment
(*n* maal onafhankelijk uitvoeren van een alternatief met een kans *p* op succes)

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}$$ waarin $0 \le x \le n$

verwachting $E(\underline{x}) = np$

Poissonverdeling:
$$P(x) = \frac{\lambda^x}{x!} e^{-\lambda}$$ verwachting $E(\underline{x}) = \lambda$

$$\sum_{x=0}^{\infty} P(x) = 1$$

Normale verdeling:
$$P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$$ verwachting $E(\underline{x}) = \mu$